A Computational Metric of the Quality of Circulation in Interior Spaces

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Abstract: Space, in terms of interior and exterior design, is one of the most important issues facing all architects. In particular the movement of people through sequences of spaces forms a large part of the circulation problem in architecture planning. Although several studies have applied network models on urban analysis to take advantage of graph based queries, understanding interior design principles based on graph attributes shows potential for further research. This paper presents a computational solution to analyse, visualize, and evaluate the circulation quality of indoor spaces. To achieve it, first we create a grid graph based on a geometrical representation of space. Using this grid, a semantic weighted graph is generated, that helps us to provide a measured score for the circulation of people in a given space. The results were tested against architects' scoring, showing that the measure is adequate. We also discuss the efficiency of our approach.

1 INTRODUCTION

Over the past few years, scientists have been applying advances in fields such as Artificial Intelligence or Computer Graphics to address multifaceted problems through intelligent applications. This is part of a growing digital revolution that has been dramatically transforming traditional disciplines. Architecture is among the most prevalent fields, and has received considerable attention from researchers, with the aim of improving the design phase and visualizing architects ideas. Recently, researchers are focusing on a new trend of design methods that exploit computational approaches to measure the quality of design elements (e.g. windows, columns, beams) from various points of view. However, there is still a need to provide strong support for architects' creativity through computerized methods, which assess the space quality.

Space, in terms of interior and exterior design, is one of the most important issues facing all architects. In this context, it can be defined as a collection of connected points satisfying particular geometric constraints. Judging the quality of a space means assessing to which extent the space configuration satisfies the expectations of the designer and the client. Spatial measurement solutions help an architect to evaluate how near each of his/her different plans are to the project objectives. A wide range of methods can be used to measure quality of space plans to obtain an appropriate view of their consequential spatial quality, before a final decision about the plan to be implemented is made. For instance, accurate statistical information could help to analyse how the configuration of architectural elements influences people's experience and behaviour. This is especially the case for large projects, involving numerous objectives, where an architect needs improved analysis tools. This analysis entails a creative consideration of all quality factors, where there is a need to determine the programmatic principals in a physical arrangement to satisfy the client's demands.

Movement patterns of people can be influenced by the perceptual thread that connects different points of the built spaces. *Circulation* is a substantial element in interior design, and architects' early designs include a relationship matrix that defines the essence of the accessibility among rooms. To support the transition from this matrix to a more creative space planning, an analytic tool of circulation that takes into account different principles of design will be needed. The principal contribution of this paper is a computational solution to analyse, visualize, and evaluate the circulation quality of indoor spaces, providing circulation scores to 3D plans, in order to help architects to decide among different designs. Our approach accepts a 3D plan and a relationship matrix as inputs. Then an algorithm extracts a grid graph at a fine level of granularity that contains all the geometrical properties of the plan. In the next step, a topological graph is generated that reveals the cost of movement among different spaces, and the traffic flow cost of a 3D plan is calculated. Finally, circulation quality is measured based on similarity of the topological graph and the relationship matrix, and the traffic score of the given 3D plan.

The remainder of the paper is organized as follows. Section 2 provides a review of some related work that attempted to measure circulation quality in architecture. Section 3 introduces our proposed approach. Section 4 discusses the experimental results for three different 3D plan. Finally, a conclusion and discussion based on our finding from this study are presented in section 5.

2 RELATED WORK

One of the earliest studies in the field uses a shared concept between architecture and geography, isovist (Benedikt, 1979), which is defined as the part of space visible from a given vantage point. The vantage point is the position of the viewer so that the quality is measured based on his/her point of view. Thus, isovist is a smart way of understanding an interior environment from the point of view of individuals, as they interact with it. This obtained visible space is associated with different measures such as area, distance, and occlusion. Kyeonah Yu (Yu, 2006) takes the advantages of isovist in path finding algorithms through a visibility graph. (Wiener & Franz, 2005) try to find out a relationship between spatial characteristics of buildings and spatial experience and behaviour of people.

Architecture is not a static experience but is experienced dynamically through circulation in the space (Puusepp, 2011). Church (Church & Marston, 2003) introduced a comparative access measurement that can be combined with traditional measures of absolute access to assist architects in making decision about finding optimized paths in urban design. Paul C Merrell et al. presented an intelligent approach for generating residential building layouts automatically (Merrell et al., 2010). Their method takes advantages of machine learning and optimization techniques for producing plausible building layouts. Although in the optimization procedure the accessibility term, along with other architectural terms, is applied for cost evaluation, it only considers the number of missing connections and entrances. Building Information Modelling (BIM) is the process of producing and managing data involving digital representations of physical and functional characteristics of a building during its life cycle. (JK Lee et al., 2008) present a BIM-enabled graph application for analysing accessible routes within indoor spaces. They use an accessible distance measurement technique and provide a visualization system highlighting spaces that are in the path. In the field of interior spaces, much work has been done to provide a spatial model for measuring the navigations quality between different space units. In addition, some studies concentrated on location-aware navigation in the form of navigation queries that help the users to find a point of interest through evaluating some factors such as travel time(Afyouni et al., 2012). According to (Afyouni et al., 2012) two types of spatial models are recognized: geometric and symbolic spatial models.

2.1 Geometrical Representation

Geometric spatial models are based on geometrical characteristics of the space. A widespread approach in the field consists of splitting the plan into certain number of non-overlapping parts. A well-known grid-based approach uses a regular tessellation method. Moravec et al (Moravec & Elfes, 1985) present high-resolution spatial maps in a system that navigates a mobile robot to a desired destination.

Although grid based approaches are appropriate for navigation and easy to implement, they are expensive in terms of memory and processing time for large spaces. This well-known geometric structure splits a space into regions close to a set of particular points of interest (H Choset, 1997).The main drawback of Voronoi tessellations is that, in some situations, the path may not be optimal (Afyouni et al., 2012).

2.2 Symbolic-Based Models

Symbolic-based approaches try to generate a graph based on topological characteristics of a given space (Dürr & Rothermel, 2003), where nodes are semantic locations (e.g., rooms, doors) and edges are connections that provide the possibility of movement between locations (Howie Choset & Burdick, 2000)(Remolina et al., 1999). *Place based* graphs are the general form of symbolic graphs where nodes are rooms and edges are doors connecting rooms. This modelling approach has been receiving much attention in navigation planning and answering nearest neighbour queries.

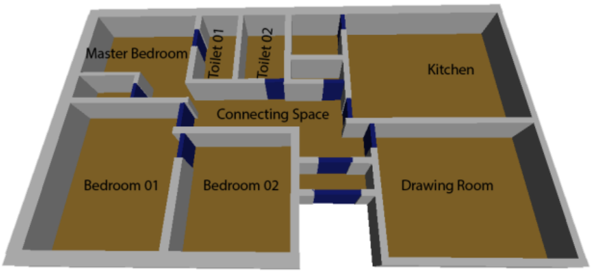

Figure 1: The input 3D plan

In (Dik-Lun et al., 2004) a semantic model is presented where the classic place-based approaches are associated with some more knowledge such as the distance between nodes. (Li et al., 2010) define a grid graph-based model of an indoor plan. The space is divided into some spatial units according to the floor plan, and then these units are represented by a grid graph where nodes and edges are labelled based on their belonging to spatial units. Their modelling approach can be applied in route, diffusion, and topological analysis.

To sum up, the common limitations of the presented methods naturally fall into one of three categories: ignoring site-specific aspects; overlooking the purpose of the building when generating semantics behind the symbolic graph; and finally the lack of a combined approach that takes advantage of both grid and symbolic graph at the same time.

3 PROPOSED APPROACH

This paper addresses how an architect can select the best (plan) among different creative design alternatives in terms of circulation functionality. While there are several guidelines for configuring architectural elements, the main motivation behind all of them is to design architectural spaces to be unobtrusive and efficient, so as to support all possible accessibility requirements. The matrix format is a commonly used method for organizing information in the pre-design stage. The density and complexity level of this matrix depends on the size and project requirements (Karlen, 2011). As pointed out above, architects often use a special type of matrix, called relationship matrix, representing relationships and adjacencies between spaces. The relationship matrix consists solely of an interpretation of accessibility information and does not propose any planning solution. Therefore, in the design process architects should comply with the expectations set out in the relationship matrix. Finding the best design solution in large projects, with a dense matrix, is typically not interesting for the analyser, and it is prone to error. In order to have an accurate understanding of accessibilities in an environment, our algorithm accepts both relationship matrix as an input as well as 3D plan that is annotated by the architect. This is a key innovation of the method we propose. Figure 1 illustrates an annotated 3D floor plan. Annotations help us to identify the functionality of each sub-space in the building. We proposed a similarity metric that measures the similarity of a symbolic matrix of a given 3D plan to the relationship matrix. In addition, several factors that are not addressed by similarity measurement, e.g. traffic and overall travel cost, are taken into consideration in measuring the circulation quality of a 3D plan.

3.1 Creating the Grid Graph

As pointed out above, the grid-based model is a well-known approach for representing navigable and impassable regions in space by assigning different labels to graph nodes. In order to create automatically a fine grid-based graph based on the geometrical attributes of 3D plan, we use a ray casting method. The granularity of the graph depends upon the partitioning complexity of the plan. Graph nodes, called *GNode*s, represent predefined places that have been extracted automatically from geometrical structure in the 3D plan. Each node has a label, for symbolic graph extraction, and at maximum 8 neighbours for navigation purposes.

First, a grid-based graph is created on top of the 3D building, according to the bounding projection of the 3D plan. Then, from each *GNode* a ray is cast down the 3D plan and, based on the collision of the ray and the 3D element inside the plan, the label of the corresponding node is determined. If the collision is detected on the wall the label is set to “impassable”, otherwise the label is assigned a value according to the spatial unit detected by a ray colliding with the building ground. As mentioned above, annotations reveal the name of each spatial unit in the 3D building, therefore these names are applied for determining label values of grid graph nodes. For instance, if a ray collides with kitchen ground, the corresponding grid node gets the label value of ‘kitchen’.

3.2 Generating the Symbolic Graph

In this step, we use a grid base graph to generate a topological (symbolic) graph that presents the possibility and cost of moving from one space to another. Nodes, called *SNodes*, symbolize predefined space landmarks extracted from *Gnode* labels. Edges stand for the weighted connections that make it possible to interact between space units (Remolina et al., 1999; Werner et al., 2000; Remolina & Kuipers, 2004). As pointed out above, *GNodes* are labelled according to their belonging to a corresponding subspace. In order to create *SNodes*, first, the *GNodes* are grouped based on their label values and then, according to each group, an *SNode* with a label corresponding to the inherited group label is created. The weight of each edge depends on the length and complexity of shortest path between two space units. Fig.3 illustrates a typical symbolic graph for the plan in figure 1.

3.2.1 Shortest path distance

Shortest path is represented by an edge whose value is the length of shortest path, in terms of number of *GNodes* in the path, between the center of a space unit corresponding to center of other space. In order to find the shortest path an A* path finding on grid graph is implemented in a way that walls are considered as impassable objects. In order to normalize the shortest path distance, we divided it by the longest possible path distance in the floor.

The longest path is a path that passes through all nodes in the grids without any duplication and ignoring impassable walls. The shortest path is calculated between two points that we calculate as the center points of the two corresponding spaces. Our definition of a center point is a point inside the space that has the minimum variation between its distances to all corner points of the space. The algorithm below describes the distance is calculated. In (1) the normalized value of shortest path is calculated.

$$NSP_{a,b} = \frac{|SP_{a,b}|}{|SP_{Max}|} \qquad (1)$$

Where $|SP_{a,b}|$ is the number of nodes in the path between *a*, *b*.

3.2.2 Path Complexity

It is generally accepted that people tend to walk along the easiest, simplest and most visible path (JK Lee et al., 2008). Human navigation pattern relies on mental planning processes which are continuously updated based on individual current perceptual configuration of the space. In doing so, we measure the complexity level of a path based on substantial factors: path visibility and direction changing.

As pointed out above, isovist measures local spatial configurations in terms of visibility from a vantage observation point. Thus, each point in the space has a particular isovist value based on its position in the space. In Figure 4 an isovist map is illustrated based on the isovist value grid points where the brighter a point is, the more isovist value it has. Of course, the more a pedestrian knows about the configuration of the space though which his walking through it, the better s/he can find his/her way. Due to the isovist quantity, we can measure the perception level of an individual at each point of the path. Therefore, by summing up the isovist value of all points in a path we can assess the quality of view point along the path. In other words, the summation value determines the simplicity level of way-finding along a given path. In (2), $NIsovist_i$ is the normalized value of Isovist of $GNode_i$ and *MaxIsovist* is the maximum value of Isovist among all *GNodes*.

$$NIsovist_i = \frac{Isovist_i}{MaxIsovist} \qquad (2)$$

Therefore in (3) $SPIsovist_{a,b}$ is the Isovist value of the shortest path between $GNode_a$ and $GNode_b$ and $|SP_{a,b}|$ is the number of *GNodes* in the path.

$$Iso_{a,b} = \frac{\sum_{i \in SP_{a,b}} NIsovist_i}{|SP_{a,b}|} \qquad (3)$$

One of the most substantial factors that affects both simplicity and visibility is the number of direction-changes through the path. In this sense, one prefers to move in a path that is as straight as possible. Hence, the more the direction of the path is changed, the more complex the path is.

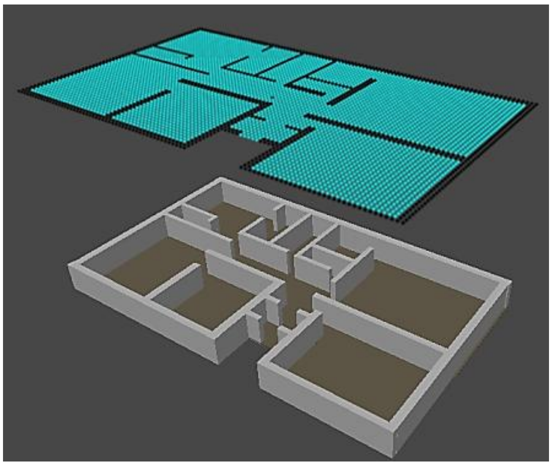

Figure 2: The generated grid-based graph.

In order to recognize when the direction is changed we use a distance measurement hypothesis. A path consists of a series of connected nodes in a way that each node, except the first on, is connected to his parent node. In order to normalize the number of direction changing we have divide it by maximum possible number of direction changing in a path. In doing so, the maximum value happens when the direction changes, approximately, in all *GNodes*.

$$NDC_{a,b} = \frac{NumOfDirCh_{a,b}}{\left|SP_{a,b}\right|} \qquad (4)$$

3.2.3 Edge Weight

The weight of the edge between *Snodes* *a* and *b* is calculated through the combination of path complexity and shortest path distance of the path that connects space unit *a* to *b*. For example in a educational building, with many students and classes, finding a shortest path is substantial while in a museum the path length is not substantial but it should cover objectives of the expedition.

$$Weight_{a,b} = (1 - NSP_{a,b})^{\lambda 1} * (Iso_{a,b})^{\lambda 2} * (1 - NDC_{a,b})^{\lambda 3} \qquad (5)$$

Where λi s adjust the weight between different terms based on the site-specific circumstances. For this paper, λi was kept at a value of 1.0

3.3 Calculating Similarity

Our similarity metric measures the similarity between two matrices: the relationship matrix and symbolic matrix.

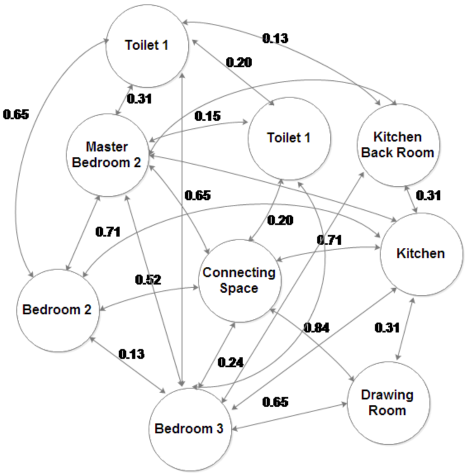

Figure 3: An example of symbolic graph of input plan.

The former is the input matrix that determines the accessibility type of space units, while the latter is the matrix representation of symbolic graph. In fact, symbolic matrix is an *s* by *s* matrix, where *s* is the number of space units. If there is a single door between space unit *a* to *b*, then the element $S_{a,b}$ is $Weight_{a,b}$, otherwise it is 0. The reason we used *weight*s instead of binary representation of the matrix is because, even if two spaces are adjacent, the door position can still have a substantial influence on the circulation pattern.

On the other hand, the input relationship matrix (or adjacency matrix) represents three levels of connectivity importance, *Must*, *Should* and *Could*, for those space units that are connected through only one door. For instance, the importance level of those spaces that are connected by *Must* is much more important than those that are connected by *Should*. For the sake of using this matrix in similarity computations, instead of qualitative terms we use three equivalent quantitative values as 1, 0.5 and 0.25 for *Must*, *Should* and *Could* respectively. Table 1 and Table 2 Illustrate a sample convert from a relation matrix R to R'. Moreover, if a plan does not satisfy even one of the *Must* conventions, the plan should be ignored. In fact, the similarity determines how much the proposed plan satisfies relationship matrix's conventions. The similarity of relationship matrix *R* and symbolic matrix *S* is calculated through (6). The more similarity, the more successful the proposed plan is in implementing relationship matrix demands.

$$PathSim_{R,S} = \frac{\sum_{i,j \in R} R[i,j] * S[i,j]}{|R|} \qquad (6)$$

3.4 Traffic

In architecture, traffic is defined as the possible number of people who are walking in a space at the same time.

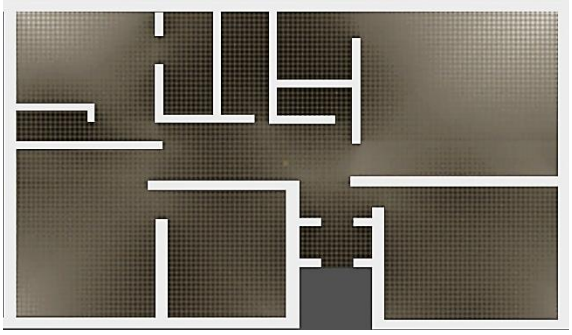

Figure 4: Isovist map of the floor plan.

Table 1: A simple relationship matrix.

R	A	B	C	D
A	0	Should	Could	Must
B	Should	0	Should	Could
C	Could	Should	0	Must
D	Must	Could	Must	0

Table 2: Quantitative representation of Matrix R.

R’	A	B	C	D
A	0	0.5	0.25	1
B	0.5	0	0.5	0.25
C	0.25	0.5	0	1
D	1	0.25	1	0

In architecture design, a *connecting space* is understood as a particular space with disjoint address spaces and a set of links connecting pairs of space units and sharing the same channel (Araújo et al., 2009). One of the most significant aspects of connecting space is the amount of possible traffic that may occur within this space. Although, increasing the size of *connecting space* can decrease the traffic, leaving a large space only for connecting space (Karlen, 2011). Therefore, architects try to consider an appropriate size with lowest traffic for connecting space. In order to measure the traffic, first we should find the connecting space in the symbolic graph. The *connecting space* is the space that has most neighbours in the symbolic graph. In (7) quality of traffic for floor plan *p* is computed.

$$Traffic_p = \frac{Area_p}{Area_{ConnectingSpace}} * \frac{NumofPath}{Area_{ConnectingSpace}} \qquad (7)$$

Where $Area_{ConncetingSpace}$ is the number of Grid nodes in the connecting space.

3.5 Overall Path Efficiency

Overall path efficiency (OPE) calculates the summation of all possible shortest path’s weights between all space units. The more summation of paths is the more efficiency can be realized for the plan’s circulation.

$$OPE = \frac{\sum_{a,b \in SpaceUnits} Weight_{a,b}}{\binom{|SpaceUnits|}{2}} \qquad (8)$$

Where the *SpaceUnits* is the set of all space units in a 3D plan.

3.6 Circulation Quality

Finally, circulation quality is calculated through weighted combination of explicit and implicit factors. The relationship matrix is determined explicitly by architect while path complexity and overall path efficiency are inferred implicitly from the 3D plan. In (9) the circulation quality of plan *P* is measured and two parameters, α and β, are defined to adjust the weight of different factors where based on the plan application. These values are defined empirically and determine the significance of each factor in measuring the quality of circulation according to building´s practices and conditions. For instance, in hospital the significance of shortest path is much higher than other parameters, therefore the value β of should be increased.

$$CQ_P = \alpha * PathSim_{R,S} + (1-\alpha) * (\beta * OPE_P + \frac{(1-\beta)}{Traffic_P}) \qquad (9)$$

4 EVALUATION

The evaluation method is defined as comparing the preferences of real architects with our generated results. The comparison process consists of presenting several different floor plans to architects and asking them to sort these design solutions based on circulation quality, then by comparing their results, we can find out how accurate our program is in satisfying architectural expectations. For this paper, a preliminary study with 6 architects was conducted. The participants were from Spain, the Netherlands, and Iran. Despite the fact that our proposed approach is more applicable in complex buildings such as hospitals and schools, to simplify the process of estimation for architects, home floor plans (instead of complex buildings’ plan) were used in this evaluation. Four floor plans, along with a relationship matrix, were presented to architects. Each of these floor plans is a design alternative that covers the expectations of relationship matrix to some extent. Each participant was asked to sort the input floor plans by considering the relationship matrix and other factors that he/she believes have influence on circulation. Participants were free to devote as much time as they need for sorting plans.

First, we sorted alternative floor plans through our proposed approach in which the output is a sorted list and $\alpha, \beta = 0.5$. Then we asked participants to sort floor plans and create a sorted list for presenting the order. Table 3 shows the results where values determine the rank of the corresponding floor plan. In order to measure the overall efficiency of our algorithm we compare the order of participants' lists with our list's order. The comparison is performed through a similarity metric that measures how closes our list is to a list that generated by a participant.

$$SimDiff_{u,v} = \frac{\sum_i |rank(u,i) - rank(v,i)|}{MaxDifference(u,v)} \quad (10)$$

Where *rank(u,i)* implies the priority of plan *i* in list *u*. In addition, *MaxDifference*(*u*,*v*) calculate the maximum possible dis-similarity between two lists *u* and *v*.

$$MaxDifference(u,v) = \sum_{i=1}^{\lceil \frac{n}{2} \rceil} (n - (2i - 1)) \quad (11)$$

Where n is the number of plans in list u that in our case is 4, thus $MaxDifference$=8.

Table 3: Evaluation results of our approach and participants

	OA	P1	P2	P3	P4	P5	P6
Plan01	2	3	2	2	3	3	3
Plan02	3	2	3	3	4	1	2
Plan03	1	1	1	4	2	2	1
Plan04	4	4	4	1	1	4	4

Table 3 shows the *SimDiff* for all participants. The result of our experiment is illustrated in Table 4. Finally, the average of *SimDiff* scores demonstrates that our approach judge the circulation quality of a plan 62 percent similar to an architect´s mind. Also, we measured the similarity between architects using the same equation. The result of this calculation was 38.3%). This lack of similarity between architects, and the higher equivalent value of our technique, suggests that our technique provides an independent method of assessing space quality that is less subject to individual bias.

Each architect spent more than 30 minutes for sorting floor plans while computation time of our algorithm is only a few minutes. We believe that in multifaceted building projects our proposed algorithm not only accelerates the decision-making process, but also assists architects to prevent errors and undesirable planning results.

5 CONCLUSIONS

Circulation is perhaps the most significant component in defining and expressing spatial form and function. Through a circulation path, a semantic relationship between spatial units is created which not only defines the quality of accessibility, but also influences other spatial quality metrics such as privacy. In this paper, we attempt to measure the circulation quality in interior spaces. The study is founded on asking ourselves how an architect can select the best solution among different creative design alternatives in terms of circulation functionality. Our proposed metric does not take into consideration changes in floor level when measuring the weight between space units. As a further line of research, it would be extremely interesting to measure the influence of floor height on path weight for those buildings containing stairs and ramps. Another promising direction is measuring the quality of circulation based in some particular situations such as hospitals and schools. In addition, we can develop this domain for analysing the quality of space according to other metrics such as privacy and illumination.

Table 4: The overall difference of our proposed approach based on *SimDiff*

	P1	P2	P3	P4	P5	P6	Avg
SimDiff	0.25	0	0.25	0.75	0.5	0.25	0.33

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